day 52

exercises whole sections

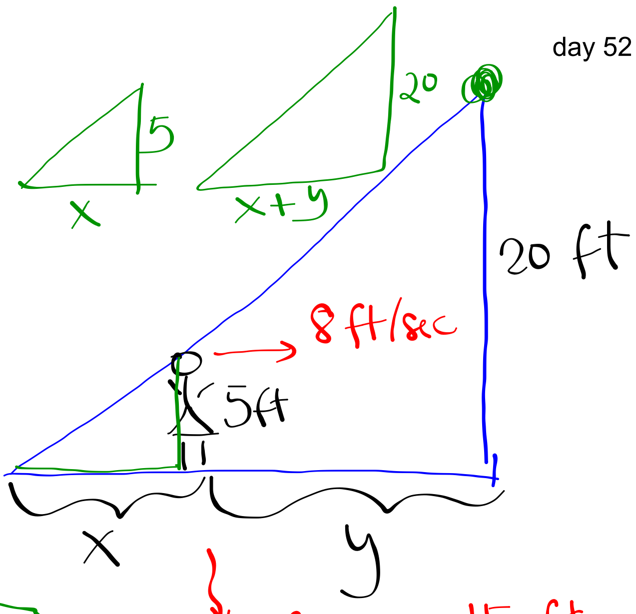
4.5 4.4

day 52

under protest - - - .

3.11/25)

(2) eqn always true



$$\frac{20}{x+y} = \frac{5}{x}$$

$$20x = 5(x+y) = 5x+5y$$

$$15x = 5y \Rightarrow 3x = y$$

how fast . . . . 15 ft

facts

$$\frac{dy}{dt} = -8 \quad f/s$$

(3) differentiate w.r.t. t

$$3\frac{dx}{dt} = \frac{dy}{dt}$$

$$3\frac{dx}{dt} = -8$$

$$\frac{dx}{dt} = -\frac{8}{3} \ (fps)$$

$$\frac{d}{dt}(3x) = 3\left(\frac{d}{dt}(x)\right) = 3\frac{dx}{dt}$$

$$\underbrace{\frac{d}{dt}(3)\cdot x}_{0} + 3\cdot\frac{d}{dt}(x) = 3\frac{dx}{dt}$$

(2b) $T = x+y$

(3b) $\frac{dT}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$

(4) $\frac{dT}{dt} = -\frac{8}{3} - 8 = -\frac{8}{3} - \frac{24}{3} = -\frac{32}{3}$

day 52

4.1/24) $f(x)=\frac{1}{8}x^3-\frac{1}{2}x$ on $[-1,3]$

critical points [[f'(x) undefined]]

[[f'(x)=0 because that's where f'(x) COULD POSSIBLY change signs]]

① $f'(x)=\frac{3}{8}x^2-\frac{1}{2}$

② $f'(x)=0 \Rightarrow \frac{3}{8}x^2-\frac{1}{2}=0$

$3x^2-4=0$

$x^2=\frac{4}{3}$ $x=\sqrt{\frac{4}{3}}, -\sqrt{\frac{4}{3}}$

only 1 in interval

critical point $\frac{2}{\sqrt{3}}$

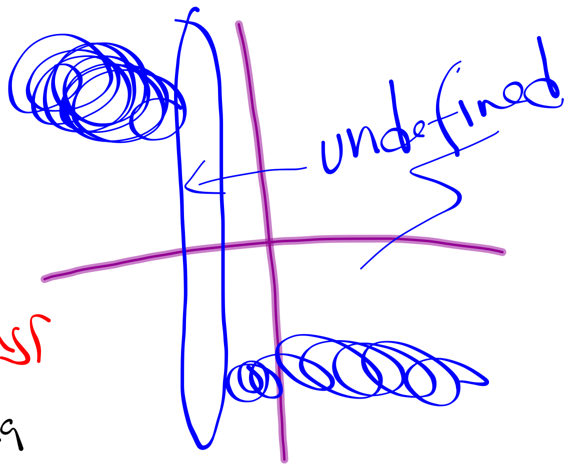
sign chart of $f'(x)$

$-\frac{1}{2}$ 0 +1

- - - - 0 + + +

-1 0 $\frac{2}{\sqrt{3}}$ 2 3

{local min}



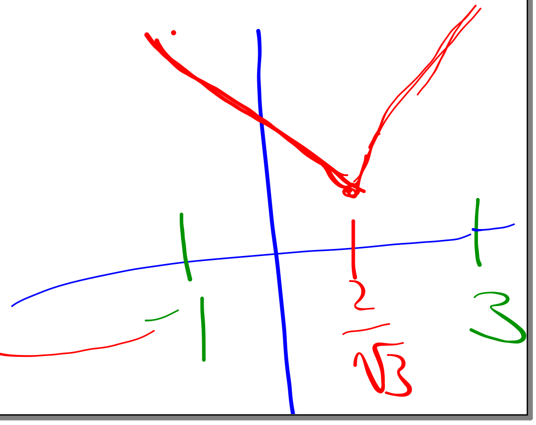
candidates for absolute things

critical pts — relative maxima

— relative minima

endpoints — $x=-1$

— $x=3$



day 52

4.1/24) $f(x) = \frac{1}{8}x^3 - \frac{1}{2}x$

1) $f'(x) = \frac{3}{8}x^2 - \frac{1}{2} = (\sqrt{3}x-2)(\sqrt{3}x+2)$

2) $f'(x) = 0$ $\quad (\sqrt{3}x-2)(\sqrt{3}x+2) = 0$

so $x = \frac{2}{\sqrt{3}}$

$x = \frac{-2}{\sqrt{3}}$

3) sign chart of $f'(x)$

$\frac{300}{8} - \frac{1}{2}$ $\quad 0 \quad -\frac{1}{2} \quad 0 \quad +1$

$+ + + \mid - - - \mid + + +$

$-10 \quad \frac{-2}{\sqrt{3}} \quad 0 \quad \frac{2}{\sqrt{3}} \quad 2$

day 52

4.1/39)

$f(x) = \cos^2 x$ on $[0, \pi]$

$f(x) = (\cos x)^2$

$f'(x) = 2(\cos x) \cdot \frac{d}{dx}(\cos x) \rightarrow (\sin x)$

$f'(x) = -2\sin x \cos x$

$f'(x) = 0$

$-2\sin x \cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2} \pm n\pi$

$\sin x = 0$

$x = 0 \pm n\pi$

$-\pi$, $-\frac{\pi}{2}$, 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$

critical pts.

from $f'(x) = 0$: $0, \frac{\pi}{2}, \pi$

from $f'(x)$ undefined: none

endpts: $0, \pi$

Candidate list: $0, \frac{\pi}{2}, \pi$

Find extrema:

$\cos^2(0) = (\cos 0)^2 = (1)^2 = 1$

$\cos^2\left(\frac{\pi}{2}\right) = \left(\cos\frac{\pi}{2}\right)^2 = (0)^2 = 0$

$\cos^2(\pi) = (\cos\pi)^2 = (-1)^2 = 1$

Absolute maximum is $y = 1$ at $x = 0, \pi$

Absolute minimum is $y = 0$ at $x = \frac{\pi}{2}$

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day 52

what is a maximum (value) of a f^n? over an interval?

The maximum of $f(x)$ is L and occurs at $x = a$ [if $f(a) = L$]

if $f(x) \leq L$ for every x

$y = 4$

$f(x) < L$

day 52

3.9/78) $f(x) = \ln\left(\frac{2x}{(x^2+1)^3}\right)$ find $f'(x)$

$f'(x) = \ln 2x - \ln(x^2+1)^3$

$\frac{1}{2x}(2) =$

$= \frac{1}{x} - \frac{d}{dx} 3 \ln(x^2+1)$

exponents apply to what is immediately preceding

$= \frac{1}{x} - 3\frac{d}{dx}\ln(x^2+1)$

$\frac{d}{dx}(cf(x)) =$
$c\frac{d}{dx}(f(x))$

$= \frac{1}{x} - (3)\frac{1}{x^2+1} \cdot \frac{d}{dx}(x^2+1)$

$= \frac{1}{x} - (3)\frac{1}{x^2+1} \cdot 2x$

$= \frac{1}{x} - \frac{3 \cdot 2x}{x^2+1}$

day 52

$$f(x) = \ln\left(\frac{2x}{(x^2+1)^3}\right) \quad \text{find } f'(x)$$

not ~~using properties of logs~~

$$f'(x) = \frac{1}{\left(\frac{2x}{(x^2+1)^3}\right)} \cdot \frac{d}{dx}\left(\frac{2x}{(x^2+1)^3}\right)$$

Quotient

$$= \frac{(x^2+1)^3}{2x}\left(\frac{2(x^2+1)^3 - 2x(3(x^2+1)^2 \cdot 2x)}{((x^2+1)^3)^2}\right)$$

day 52

HW/ 4.1/ 40-43, 56-58

4.2/ 11, 15, 17-20, 27-30, 39-40

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